Wage Negotiations in Multi-Worker Firms and Stochastic Bargaining Powers of Existing Workers*

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November 2020

Abstract

This paper develops a business cycle search and matching model in which the volatility of employment closely matches that of the U.S. data without wage rigidity and the labor share overshoots in response to productivity shocks. To this end, I introduce an alternative mechanism of wage negotiations and bargaining shocks in an environment where a firm hires more than one worker and the firm faces diminishing marginal product of labor (MPL). When Nash bargaining with a marginal worker breaks down, a firm negotiates wages with existing workers and produces with them. Due to diminishing MPL, the breakdown in the negotiation with the marginal worker negatively affects the bargaining position of the firm with existing workers (one fewer workers) since MPL is higher with one fewer workers. How much the firm internalizes this negative effect depends on stochastic bargaining powers of existing workers which can be identified through labor share data. The stochastic bargaining power of existing workers provides an additional margin to increase the volatility of labor market variables including wages.

Keywords: business cycle, wage negotiation, multi-worker firms, existing workers, bargaining shocks

JEL Classification Numbers: E24, E32, E60, H55, I38, J65

*I am grateful to Jonathan Heathcote and José-Víctor Ríos-Rull. I also would like to thank Yongsung Chang, Jong-Suk Han, Yong Jin Kim, John Seliski, Naoki Takayama as well as seminar participants at the SED meeting in Seoul, the University of Minnesota, Chung-Ang University, Myongji University, and Yonsei University for helpful comments. All errors are mine. This work was supported by the Hongik University new faculty research support fund. An earlier version of this paper was circulated under the title "Bargaining with Existing Workers, Overhiring of Firms, and Labor Market Fluctuations".

1 Introduction

This paper develops a business cycle search and matching model in which the volatility of employment closely matches that of the U.S. data without wage rigidity and the labor share overshoots in response to productivity shocks. In literature, two different bargaining protocols are used in the search and matching model where a firm hires more than one worker and the firm faces diminishing marginal product of labor (MPL). One is the Stole and Zwiebel (1996) type bargaining protocol as in Elsby and Michaels (2013), Acemoglu and Hawkins (2014), Hawkins (2015), Dossche et al. (2018), and Kudoh et al. (2019). In these papers, a breakdown in a negotiation with a marginal worker negatively affects the bargaining position of the firm with other workers (one fewer workers) since MPL is higher with one fewer workers. The other is a standard bargaining protocol as in Merz (1995), Andolfatto (1996), and Cheron and Langot (2004). In these papers, a breakdown in a negotiation does not affect the bargaining with other workers because they implicitly assume that MPL does not change when the firm bargains with other workers. I interpret these two bargaining protocols as two extreme cases: in terms of relative bargaining powers between a firm and other workers. I will call other workers existing workers. If existing workers have all the bargaining powers[1], then the firm has to fully internalize the negative effects from the breakdown in the negotiation with a marginal worker. However, if the firm has all the bargaining powers[2], the firm does not internalize any negative effects from the breakdown by ignoring that MPL is higher with one fewer workers. Given the two extreme cases, I am looking at cases between the two extremes by introducing stochastic bargaining powers of existing workers.

In this paper, when Nash bargaining with a marginal worker breaks down, a firm negotiates wages with existing workers. The bargaining power of existing workers differs from the bargaining power of the marginal (or new) worker and moves stochastically. Due to diminishing MPL, the breakdown in the negotiation with the marginal worker negatively affects the bargaining position of the firm with existing workers (one fewer workers) since MPL is higher with one

[1] This is my interpretation of the Stole and Zwiebel type bargaining protocol.
[2] This is my interpretation of the standard bargaining protocol.

fewer workers. How much the firm internalizes this negative effect depends on the stochastic bargaining powers of existing workers which can be identified through labor share data. During expansions, hiring workers is relatively difficult for firms, and thus, existing workers have relatively higher bargaining power. If the firm fails to hire a marginal worker due to a breakdown in negotiations, then the firm has to pay higher wages to existing workers. Considering that the failure to hire marginal workers is more costly during expansions, the firm has more incentive to hire marginal workers by offering higher wages (and hours per worker) to forgo the higher cost associated with the breakdown. The stochastic bargaining power of existing workers amplifies this mechanism. The bargaining power of existing workers increases more during expansions through pro-cyclical bargaining shocks in addition to an improvement in the outside option of existing workers that occurs even when a fixed bargaining power of existing workers is assumed. During recessions, the opposite happens. Through this mechanism, the stochastic bargaining powers of existing workers provide an additional margin to increase the volatility of labor market variables. The stochastic bargaining power of existing workers simultaneously increases the volatility of employment, hours per worker, and wages. This mechanism does not rely on wage rigidity. Mechanically, the bargaining power of existing workers affects the outside option value for firms (i.e., cost associated with a breakdown in negotiations with marginal workers) in this study. The pro-cyclical bargaining power of existing workers results in the counter-cyclical firm's outside option and the pro-cyclical firm's surplus. Consequently, the pro-cyclical bargaining power of existing workers leads to more flexible wages. The calibrated model generates more volatile total hours, employment, hours per worker, and wages while labor share overshoots in response to productivity shocks as documented in Ríos-Rull and Santaeulalia-Llopis (2010). In particular, the volatility of employment in the model is similar to the actual U.S. data. In contrast to the prediction of Ríos-Rull and Santaeulalia-Llopis (2010), in which the effect of productivity shocks is dampened when labor share overshoots due to huge wealth effects from the overshooting property of labor share, this paper presents a model in which the labor share overshoots in response to productivity shocks and the volatility of employment closely matches that of the U.S. data without wage rigidity.

This paper is related to several studies which can be classified into three groups. First, the baseline model is based on Andolfatto (1996). His model embeds search and matching framework into an otherwise standard RBC model, and has both extensive margins and intensive margins. By incorporating search and matching framework in labor markets, the model improves the standard RBC model along several dimensions. However, the volatility of labor market variables is still far lower than that of actual data. The Andolfatto model also has highly pro-cyclical real wages and labor productivity, which have weakly pro-cyclical counterparts in actual data. Several papers have addressed these problems. Nakajima (2012) analyzes several volatility problems by explicitly distinguishing between leisure and unemployment benefits for the outside options of households. This distinction is consistent with the calibration proposed by Hagedorn and Manovskii (2008). However, the main focus of Nakajima (2012) is unemployment and vacancies than employment and hours per worker, which are my main interest. Cheron and Langot (2004) address the second failure of Andolfatto (1996) by using non-separable preference between consumption and leisure such that the outside options of households can move counter-cyclically. This proposal results in less pro-cyclical real wages and labor productivity. However, this paper is not interested in the volatility of labor market variables in general.

The second branch of papers related to my study is the literature on the Stole and Zwiebel bargaining and its application to macroeconomics. Cahuc and Wasmer (2001), Ebell and Haefke (2003), and Cahuc et al. (2008) first incorporate the Stole and Zwiebel bargaining into a search and matching model in macroeconomics.[3] Recently, there are several studies on the application

[3]Cahuc and Wasmer (2001) integrate the Stole and Zwiebel bargaining protocol into the standard search and matching model with large firms. They demonstrate that the standard matching model with large firms is fully compatible with the Stole and Zwiebel bargaining under the assumptions of constant returns to scale and costless capital adjustment. This finding also implies that the prediction of search and matching models with the Stole and Zwiebel bargaining may differ from that of the model with the standard Nash bargaining under the assumptions of decreasing returns to scale or costly capital adjustment. Ebell and Haefke (2003) study the relationship between product market regulation and labor market outcomes in a search and matching model with imperfect competition and the Stole and Zwiebel bargaining. They find that the standard monopoly distortion of underproduction is partially offset by overhiring from the Stole and Zwiebel bargaining, particularly when monopoly power is high. Cahuc et al. (2008) provide explicit closed form solutions in a search and matching model with heterogeneous workers and the Stole and Zwiebel bargaining. They demonstrate that firms can underemploy workers with a relatively lower bargaining power to overemploy workers with a relatively higher bargaining power.

of the Stole and Zwiebel type bargaining to business cycle dynamics.[4] Krause and Lubik (2013) incorporate the Stole and Zwiebel type bargaining protocol into a simple RBC search and matching model to evaluate the quantitative effects of the bargaining protocol on business cycle dynamics. They show that the aggregate effects of the bargaining protocol on business cycle moments are negligible. In contrast to Krause and Lubik (2013), this paper introduces the stochastic bargaining with existing workers when the match with a marginal worker fails, and the bargaining powers of existing workers vary stochastically. The time-varying incentive to hire workers for firms, resulting from the stochastic bargaining powers, provide a new margin to increase the volatility of labor market variables.[5] Dossche et al. (2018) and Kudoh et al. (2019) examine the labor market effects of variable hours per worker (intensive margin) with the Stole and Zwiebel type bargaining. Dossche et al. (2018) find that the overhiring is larger when firms can bargain hours per worker as well as employment with workers because hiring new workers decreases hours per worker for existing workers and hence lowers values of the outside options and equilibrium wages for the workers. Kudoh et al. (2019) show that when hours per worker are not negotiated and are determined by firms, labor market fluctuations can be amplified even with a small Frisch elasticity. Their model can successfully explain the labor market fluctuations in Japan.

Lastly, this paper is also related to papers studying labor share dynamics. Ríos-Rull and Santaeulalia-Llopis (2010) document several properties of labor share dynamics based on the U.S. data. In particular, they propose redistributive shocks that can be identified by using labor share data in the US, and point out the importance of the dynamic property of labor share. They showed that labor share overshoots in response to productivity shocks (overshooting property), and the dynamic overshooting response of labor share drastically dampens the role of productivity shocks on labor markets due to huge wealth effects. My model also generates the overshooting property of labor share, but total hours, employment, and hours per worker

[4] Elsby and Michaels (2013), Acemoglu and Hawkins (2014), and Hawkins (2015) also study the labor marker fluctuations with the Stole and Zwiebel type bargaining, but the main focus of these papers is unemployment and vacancies than employment and hours per worker.

[5] Later, it turns out that Andolfatto (1996) and Krause and Lubik (2013) are two extreme cases where bargaining powers of existing workers are fixed at 0 and 1 respectively in the baseline model.

are still more volatile than the benchmark Andolfatto model. Different from Rios-Rull and Santaeulalia-Llopis (2010), the search and matching framework weakens wealth effects from the overshooting of labor share and the pro-cyclicality of incentive for firms to hire workers is amplified by bargaining shocks and hence offsets the huge reduction of total hours. Colciago and Rossi (2015) and Mangin and Sedlacek (2018) develop search and matching model can generate the counter-cyclicality and overshooting property of the labor share. Colciago and Rossi (2015) reflect strategic interactions among an endogenous number of producers in the model, which leads to counter-cyclical price markup. In order to improve the labor share dynamics in the model, Mangin and Sedlacek (2018) introduce heterogenous firms competing to hire workers and investment-specific technology shocks. The main focus of these two papers is labor share itself than other labor market variables such as employment and hours per worker.

The main contribution of this paper is as follows. First, this paper presents a model in which the volatility of employment closely matches that of the U.S data without wage rigidity by incorporating the stochastic bargaining power of existing workers into the Andolfatto model.[6] Jung and Kuester (2015) assume the stochastic bargaining power of marginal workers (not exisiting workers). The counter-cyclical bargaining power of marginal workers results in more rigid wages by construction and the amplification of labor market fluctuations.[7] By contrast, this paper introduces the pro-cyclical bargaining power of existing workers (not marginal workers) resulting in more flexible wages relative to the standard business cycle model with search frictions, such as that of Andolfatto (1996). Despite the more flexible wages, the volatility of labor market variables increases. The bargaining powers of existing workers can be time-varying because when labor markets are tighter, mostly in booms, existing workers are more valuable as the firm will have difficulty finding new workers. However, when the labor market

[6] The Andolfatto model has both extensive and intensive margins in the labor marketI include intensive margins for two reasons. First, labor share is important for identifying bargaining shocks. Therefore, for the labor share in the model to be consistent with actual data, I need to include intensive margins. Second, bargaining shocks directly affect intensive margins because the bargaining powers of existing workers affect the relative usefulness of intensive and extensive margins for the firm as noted in Dossche et al. (2018) and Kudoh et al. (2019).

[7] The counter-cyclical bargaining power of marginal workers reduces wage volatility because negotiated wages are decreased during expansions and increased during recessions. Their calibration entails considerable wage rigidity in order to match the amplitude of labor-market fluctuations observed in the data.

is less tight, mostly in recessions, existing workers become less attractive to firms, which could easily find new workers. This reason makes the bargaining powers of existing workers possibly pro-cyclical with some lags. The stochastic bargaining power of existing workers amplifies the pro-cyclicality. The inclusion of the stochastic bargaining with existing workers improves the capacity of the standard RBC search and matching model, especially in the volatility of total hours, employment, hours per worker, wages and labor share.

Second, my model generates an overshooting property of labor share which is driven by exogenous bargaining shocks, but the effect of productivity shocks on labor market variables are still significant in contrast to the prediction of Ríos-Rull and Santaeulalia-Llopis (2010). In their model, the effect of productivity shocks is dampened when labor share overshoots because of huge wealth effects from the overshooting property. In contrast to their model, the baseline model has a search and matching framework, and the nature of this framework weakens wealth effects resulting from the overshooting of labor share. On top of these differences, more incentive for firms to hire workers due to bargaining shocks offset the huge reduction of total hours in booms.

The remainder of the paper is structured as follows. Section 2 introduces the baseline model with the stochastic bargaining powers of existing workers. Section 3 discusses the calibration of the baseline model. Section 4 shows quantitative analysis of the model. Section 5 examines the robustness of the baseline model. Section 6 discusses the implications of the paper for the trend decline in labor share. Finally, Section 7 concludes and proposes the further research.

2 Model

I develop a model based on a standard RBC search and matching model, the Andolfatto (1996) model. The main difference between the model in this paper and the Andolfatto model is the outside option of a firm in the bargaining with a marginal worker. I explicitly consider the outside option of a firm when the firm bargains with a marginal worker. The outside option of the firm is bargaining with existing workers (one fewer workers) and producing goods with them. The issue with bargaining with existing workers is the wages the firm pays. In this paper, these wages depend on the bargaining powers of existing workers.[8] If the bargaining powers of existing workers are high, then existing workers will receive higher wages, but if the bargaining powers of existing workers are low, then they will receive lower wages. Note that these wages are not realized if the match with the marginal worker is successful while they still affect the equilibrium wages. In this paper, matches are always successful because the surplus of a new match is always positive in the calibrated model. Therefore, wages bargained with existing workers are not be realized in equilibrium. Furthermore, I assume the bargaining powers of existing workers stochastically evolves. Except for the stochastic bargaining with existing workers, the baseline model is similar to Andolfatto (1996) and Cheron and Langot (2004).

2.1 Matching

I assume that the period in the model is a quarter. The timing of my model is as follows: (1) shocks are realized, (2) wages and hours per worker are bargained over with marginal workers, (3) if matches are not successful, the firm bargains wages with existing workers (4) workers are matched with the firm (5) production takes place and the firm posts vacancies, and (6) exogenous separations occur.

Since labor markets are frictional, the unemployed search for jobs and firms post vacancies to hire workers. The number of matches is determined by constant returns to scale matching function $M = M(V, 1 - N)$, which depends on the total number of vacancies (V) and the total number of the unemployed ($U \equiv 1 - N$). For later use, I define $\theta = V/(1 - N)$ as market tightness in labor markets. Also, I define the job-finding rate $p(\theta) \equiv M/(1 - N) = M(\theta, 1)$ and the job-filling rate $q(\theta) \equiv M/V = M(1, 1/\theta)$. Finally, I assume that workers are separated at the exogenous and constant rate $\chi \in (0, 1)$. Therefore, we have the following law of motion of total employment.

[8] As already stated, the bargaining power of existing workers is different from the bargaining power of marginal workers.

$$N^{'} = (1-\chi) N + M(V, 1-N)$$

2.2 Household

There is a continuum of identical and infinitely lived households of measure one. The measure of members in each household is also normalized to one. A representative firm is owned by the household. The aggregate states in this economy are given by $S = \{z, \gamma; K, N\}$, where z is the aggregate productivity and γ is the bargaining power of existing workers, which varies stochastically. K is the aggregate capital stock and N is total employment in the economy. The individual state variables of the household are $s_H = \{a, n\}$, where a is the amount of assets they hold and n is the measure of the employed in household. I can write the household problem as follows:

$$\Omega(S, s_H) = \max_{c,a^{'}} u(c) + n\widetilde{u}(1 - h(S, s_H)) + (1-n)\widetilde{u}(1) + \beta E\left[\Omega\left(S^{'}, s_H^{'}\right)\right] \tag{1}$$

$$\begin{aligned} &s.t. \\ c + a^{'} + T(S) &= nw(S, s_H) h(S, s_H) + (1-n) b + (1 + r(S)) a + \Pi(S) \\ n^{'} &= (1-\chi) n + p(S)(1-n) \\ S' &= G(S) \end{aligned}$$

where $u(c)$ is utility from consumption, a is the assets household holds, $\widetilde{u}(\cdot)$ is utility from leisure, $T(S)$ is the lump-sum tax, $\Pi(S) = F(z, k, nh) - w(S, s_F) h(S, s_F) n - (r(S) + \delta) k - \kappa v$ is the dividend which will be defined in the firm's problem. $p(S) = M/(1-N)$ is the job-finding rate and $G(S)$ is the law of motion of aggregate state variables. Household takes wages $(w(S,\, s_H))$ and hours per worker $(h(S, s_H))$ as given. They are jointly determined via Nash bargaining.

The household consumes (c), accumulate assets (a) which they rent to a firm, and supplies labor. The n fraction of members in each household is matched with the firm and employed. And the $1-n$ fraction of members is unemployed, searches for jobs, and they collect unemployment benefits (b) from the government. I assume that there is no search cost, and so every member who is not employed searches for the job.[9] I also assume that there is a perfect insurance for unemployment within the household as noted in Andolfatto (1996).[10] As a result, every member receives the same consumption level. Note that this implies unemployed members are better off than those who are employed since they receive the same consumption level but the unemployed enjoy a full amount of leisure.[11]

The first order conditions of household's problem give[12]

$$\beta E\left[\frac{u_c^{'}}{u_c}\left(1+r^{'}\right)\right] = 1 \tag{2}$$

This is a standard Euler equation for the household.

2.3 Firm

There exists a representative firm. The firm produces goods using a constant returns to scale production technology $F(z,k,nh)$, where z is the aggregate productivity. Given the aggregate state S and the individual state variable of the firm $s_F = \{n\}$, I can write firm's recursive problem as follows:

$$\begin{aligned} J(S,s_F) &= \max_{v,k} \Pi(S) + E\left[\widetilde{\beta}(S,S')\, J(S',s_F')\right] \qquad (3) \\ &= \max_{v,k} F(z,k,nh) - w(S,s_F)\, h(S,s_F)\, n - (r(S)+\delta)\, k - \kappa v + E\left[\widetilde{\beta}(S,S')\, J(S',s_F')\right] \\ s.t. & \\ n' &= (1-\chi)\, n + q(S)\, v \\ S' &= G(S) \end{aligned}$$

where $\widetilde{\beta}\left(S,S^{'}\right) = \beta u_c\left(c\left(S^{'}\right)\right)/u_c\left(c\left(S\right)\right)$ is the stochastic discount factor, κ is the cost of posting vacancies, and $q(S) = M/V$ is the job-filling rate. Again, $G(S)$ is the law of motion

[9] In this sense, u in my model is the non-employed. I do not distinguish between the unemployed and the non-employed following Andolfatto (1996). Since the measure of the unemployment rate in model and data are inconsistent, I do not report any statistics regarding unemployment in this paper.

[10] Separable utility functions over consumption and leisure satisfy this assumption.

[11] I can relax this assumption. As noted in Cheron and Langot (2004), Nakajima (2012), if I use non-separable utility functions over consumption and leisure, the employed receive higher levels of consumption than the unemployed. Consequently, the employed are better off in equilibrium. If I use non-separable utility functions, the performance of the model would be better, especially for labor productivity and real wages. However, I do not use these utility functions because I prefer to setup the baseline model in a more parsimonious way to make a fair comparison of my model and Andolfatto (1996).

[12] I will drop state variables for simple notations.

of aggregate state variables. The firm hires workers and rent capital from the households, and posts vacancies to hire more workers in the next period. Firms also take wages ($w(S, s_F)$) and hours per worker ($h(S, s_F)$) as given. They are jointly determined via Nash bargaining. From the first order conditions, we have two equilibrium conditions.

$$
\begin{aligned}
r &= F_k - \delta && (4) \\
\kappa &= qE\left[\widetilde{\beta} J^{m'}\right] && (5)
\end{aligned}
$$

where J^m is a marginal value of an additional employee to the firm. The first condition is an equation for the equilibrium rental rate. The second equation is a job creation condition, which implies the firm posts vacancies up to the point where the marginal cost of posting vacancies equals to the value of an additional worker discounted by the probability that the firm meets a marginal worker.

2.4 The bargaining with a marginal worker

As stated before, wages (w), and hours per workers (h) are jointly determined via Nash bargaining between a worker and a firm each period. Since there is no heterogeneity among workers, every worker is treated as a marginal worker. Formally, Nash bargaining with a marginal worker can be written as follows:

$$
\begin{aligned}
(w, h) &= arg \max_{w,h} (\Omega^m)^{\mu} (J^m)^{1-\mu} \\
&= arg \max_{w,h} \left(\frac{V^E - V^U}{u_c}\right)^{\mu} \left(\lim_{\Delta \to 0} \frac{J[n+\Delta] - J^B[n]}{\Delta}\right)^{1-\mu} \qquad (6)
\end{aligned}
$$

The first component (Ω^m) denotes the marginal value of employment for the worker[13] and the second component (J^m) represents the marginal value of an additional employee to the firm. μ is the bargaining power of a marginal worker.[14] V^E is the value of employment for the worker and V^U is the value of unemployment for the worker, which is the outside option of the worker. $J[n+\Delta]$ is the value of the firm when the match with the $(n+\Delta)$-th worker is successful and

[13]Note that this value is discounted by the marginal utility of consumption so that the unit of this term can be converted to consumption goods.

[14]The bargaining power of marginal workers (μ) differs from that of existing workers (γ) and is assumed to be a constant.

$J^B[n]$ is the value of the firm when the negotiation breaks down, which is the outside option of the firm. The only difference between the bargaining problem in this paper and the standard Nash bargaining is the outside option of the firm ($J^B[n]$), which is defined within the marginal value of an additional employee to the firm (J^m).

2.4.1 The marginal value of employment for the worker

I can define the marginal value of employment for the worker as follows.

$$\begin{aligned}
\Omega^m = \frac{V^E - V^U}{u_c} &\equiv \frac{1}{u_c}\left[whu_c + \widetilde{u}(1-h) + (1-\chi)\beta E\left[V^{E'}\right] + \chi\beta E\left[V^{U'}\right]\right] \\
&\quad -\frac{1}{u_c}\left[bu_c + \widetilde{u}(1) + p\beta E\left[V^{E'}\right] + (1-p)\beta E\left[V^{U'}\right]\right] \\
&= wh - b - \frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + (1-\chi-p)\beta E\left[\frac{V^{E'} - V^{U'}}{u_c}\right] \\
&= wh - b - \frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + (1-\chi-p)\beta E\left[\frac{u_c'\Omega^{m'}}{u_c}\right]
\end{aligned} \tag{7}$$

Note that the bracket in the first line is the value of working which includes utility from consumption, utility from leisure, and the continuation value of employment for the worker. The bracket in the second line is the outside option of the worker which consists of utility from consumption, utility from leisure, and the continuation value of unemployment for the worker. From the value function of the household, we also have

$$\frac{\Omega_n}{u_c} = wh - b - \frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + (1-\chi-p)\beta E\left[\frac{\Omega_n'}{u_c}\right] \tag{8}$$

From Equation (7) and (8), we have

$$\Omega^m = \frac{\Omega_n}{u_c} \tag{9}$$

Therefore, the marginal value of employment for the worker that I defined before is the same as the partial derivative of the value function of the household with respect to the number of the employed in household (n).

2.4.2 The marginal value of an additional employee to the firm

The marginal value of an additional employee to the firm is not trivial because the outside option of the firm can be defined in different ways. The outside option of the firm in the bargaining with a marginal worker is bargaining wages with existing workers and producing goods with them. The key component of the outside option for the firm is the wages the firm pays to existing workers. Let w^e be the wages negotiated between the firm and existing workers when the match with a marginal worker breaks down.[15] I can define the value of an additional employee to the firm (J^m) as follows:

$$\begin{aligned}
J^m &\equiv \lim_{\Delta\to 0}\frac{J\left[n+\Delta\right]-J^B\left[n\right]}{\Delta} \\
&= \lim_{\Delta\to 0}\frac{1}{\Delta}\left[\left(F(z,k,(n+\Delta)\,h)-w\left[n+\Delta\right](n+\Delta)\,h-(r+\delta)\,k-\kappa v+\beta E\left[\frac{u'_c}{u_c}J\left[(n+\Delta)'\right]\right]\right)\right. \\
&\quad \left.-\left(F(z,k,nh)-w^e\left[n\right]nh-(r+\delta)\,k-\kappa v+\beta E\left[\frac{u'_c}{u_c}J^B\left[n'\right]\right]\right)\right] \\
&= F_n-\lim_{\Delta\to 0}\frac{w\left[n+\Delta\right](n+\Delta)\,h-w^e\left[n\right]nh}{\Delta}+(1-\chi)\,\beta E\left[\frac{u'_c}{u_c}J^{m'}\right] \qquad (10)
\end{aligned}$$

$J\left[n+\Delta\right]$ denotes the value of the firm when the match with a marginal worker is successful.[16] $J^B\left[n\right]$ denotes the value of the firm when the negotiation with the marginal worker breaks down, which is the outside option of the firm. When the bargaining with the marginal worker breaks down, the firm continues to produce goods with existing workers by continuing wage negotiations with existing workers afterward. $w\left[n+\Delta\right]$ is Nash bargained wages with the $(n+\Delta)$-th worker and $w^e\left[n\right]$ is wages for existing workers when the match breaks down. The second line is the value of the firm when the firm hires $\triangle$ more workers, which includes the level of output less wage bills with workers including newly hired ones and costs of posting vacancies, and the continuation value of the firm. The third line is the outside option of the firm, which consists of the level of output less wage bills with existing workers and costs of posting vacancies, and the continuation value of the firm. The derivation of the last equation can be found in the Appendix. If the firm has all the bargaining powers, then the firm does

[15] As stated before, the wages for existing workers (w^e) are not be realized in equilibrium. The wages show up in the outside option of the firm, but the match with a marginal worker is always successful in this paper because the match surplus is always positive in the calibrated model. Consequently, the wages for existing workers are not realized in equilibrium while they still affect equilibrium wages and other variables.

[16] I drop aggregate state variables for simple notations here.

not internalize any negative effects from the breakdown of the negotiation with the marginal worker by ignoring that MPL is higher with one fewer workers. In this case, it is assumed that the firm pays existing workers the same wages as the firm would have paid the marginal worker. Then, we have $w^e[n] = w[n+\Delta]$.

Proposition 1

Suppose $w^e[n] = w[n+\Delta]$. Then, the marginal value of an additional employee to the firm reduces to

$$J^m = F_n - wh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right] \tag{11}$$

Proof. See the Appendix. □

This is the standard marginal value of an additional employee to the firm in literature where wages are determined via the standard bargaining protocol as in Merz (1995), Andofatto (1996), and Cheron and Langot (2004). Also, note that this equation can be directly derived by differentiating the firm's value function (J) with respect to n, under the assumption that wages are not a function of n. On the other hand, if existing workers have all the bargaining powers, then the firm should fully internalize the negative effects from the breakdown. In this case, it is assumed that the firm continues Nash bargaining with $\triangle$ fewer workers and we have $w^e = w[n]$, where $w[n]$ is Nash bargained wages with n-th worker.

Proposition 2

Suppose $w^e[n] = w[n]$. Then, the marginal value of an additional employee to the firm reduces to

$$J^m = F_n - w[n]h - \frac{\partial w[n]}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right] \tag{12}$$

Proof. See the Appendix. □

This is the marginal value of an additional employee to the firm when wages are determined via the Stole and Zwiebel bargaining protocol as in Elsby and Michaels (2013), Acemoglu and Hawkins (2014), and Hawkins (2015). Note that this equation can be directly derived by differentiating the firm's value function (J) with respect to n, under the assumption that wages are an explicit function of n. The partial derivative term ($-\frac{\partial w[n]}{\partial n}$) captures the additional value of hiring workers in the sense that hiring new workers lowers the wages of existing workers. The sign of the term ($\frac{\partial w[n]}{\partial n}$) will be turned out to be negative later in the calibration section.

In this paper, the wages for existing workers ($w^e[n]$) depend on the bargaining powers of existing workers (γ). More specifically, it is assumed $w^e[n] \equiv \gamma w[n] + (1-\gamma) w[n+\Delta]$. For example, if existing workers have higher bargaining powers, they receive wages closer to $w[n]$, and if they have lower bargaining powers, they receive wages closer to $w[n+\Delta]$.

Proposition 3

Suppose $w^e[n] = \gamma w[n] + (1-\gamma) w[n+\Delta]$. Then, the marginal value of an additional employee to the firm reduces to

$$J^m = F_n - w[n]h - \gamma \frac{\partial w[n]}{\partial n} nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c} J^{m'}\right] \tag{13}$$

Proof. See the Appendix. □

By construction, if $\gamma = 0$, then *Proposition 3* reduces to *Proposition 1* (standard bargaining protocol), and if $\gamma = 1$, then *Proposition 3* reduces to *Proposition 2* (Stole and Zwiebel bargaining protocol). Note that the marginal value of an additional employee to the firm depends on the stochastic bargaining power of existing workers through the term ($-\gamma\frac{\partial w[n]}{\partial n}nh$). This is the main contribution of this paper. The inclusion of the stochastic bargaining with existing workers provides an additional margin to increase the volatility of labor market variables basically through the term ($-\gamma\frac{\partial w[n]}{\partial n}nh$) that appears within the the marginal value of an additional employee to the firm (J^m).

2.4.3 Stochastic bargaining powers of existing workers (γ)

The bargaining power of existing workers (γ) can be time-varying because when labor markets are tighter, mostly in booms, existing workers are more valuable as the firm will have difficulty finding new workers. However, when the labor market is less tight, mostly in recessions, existing workers become less attractive to firms, which could easily find new workers. This reason makes the bargaining power of existing workers possibly pro-cyclical with some lags.

Since the baseline model does not have any endogenous mechanism to generate time-varying bargaining power of existing workers, I will assume that $\gamma \in [0, 1]$ varies stochastically and call innovations to γ bargaining shocks. I will show, in the calibration section, that bargaining shocks can be identified by using labor share data from US once we have the solution to the first order differential equation from the wage bill equation. It is assumed that the bargaining power for a marginal worker (μ) is fixed while the bargaining power of existing workers (γ_t) varies over time. In the robustness section, I show that the time-varying bargaining power of a marginal worker is quantitatively not an important factor given the constructed shock series of bargaining power of a marginal worker (μ_t) by using labor share data. I will discuss it more in detail in the robustness section.

2.4.4 Solutions to the bargaining with a marginal worker

Now, we turn to the bargaining problem which is the same as standard Nash bargaining given the marginal value of employment for the worker and the marginal value of an additional employee to the firm.

$$\Omega^m = wh - b - \frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + (1-\chi-p)\beta E\left[\frac{u'_c}{u_c}\Omega^{m'}\right] \tag{14}$$

$$J^m = F_n - wh - \gamma\frac{\partial w}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right] \tag{15}$$

Given the bargaining power of the marginal worker ($\mu \in [0, 1]$) and the bargaining powers of existing workers ($\gamma \in [0, 1]$), wages and hours per worker are determined via the following standard bargaining problem

$$(w,h) \quad = \quad arg \max_{w,h} (\Omega^m)^{\mu} (J^m)^{1-\mu} \tag{16}$$

I will write w instead of $w[n]$ for simple notations hereafter. From the first order conditions with respect to w and h, we have the following two equations.

$$wh \quad = \quad \mu\left(F_n - \gamma\frac{\partial w}{\partial n}nh + \frac{V}{1-N}\kappa\right) + (1-\mu)\left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b\right) \tag{17}$$

$$\frac{\widetilde{u}_l(1-h)}{u_c} \quad = \quad F_{nh} - \gamma\frac{\partial w}{\partial n}n \tag{18}$$

where $F_{nh} = \frac{\partial F(z,k,nh)}{\partial(nh)}$.[17] Equation (17) is the wage bill equation and Equation (18) is an intra-temporal condition for hours per worker. Note that we have additional terms, $-\gamma\frac{\partial w}{\partial n}nh$ and $-\gamma\frac{\partial w}{\partial n}n$ in Equation (17) and (18) compared to the standard bargaining case. The term $\frac{\partial w}{\partial n}$ can be calculated by solving the first order differential equation, which will be defined from the wage bill equation shortly. Equation (17) is similar to the wage bill equation as in Cheron and Langot (2004) except for the second term in the right hand side, $-\gamma\frac{\partial w}{\partial n}nh$. I can rewrite the wage bill equation as the first order differential equation with respect to wages w. Assuming a Cobb-Douglas production function, $F(z,k,nh) = e^z k^{\alpha}(nh)^{1-\alpha}$, the solution to the first order differential equation is given as

$$w \quad = \quad \mu\left(\frac{1-\alpha}{1-\mu\gamma\alpha}e^z k^{\alpha} h^{-\alpha} n^{-\alpha} + \frac{V}{1-N}\kappa h^{-1}\right) + (1-\mu)\left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b\right)h^{-1} \tag{19}$$

From Equation (19), we have

$$\frac{\partial w}{\partial n} \quad = \quad -\frac{\mu\alpha(1-\alpha)}{1-\mu\gamma\alpha}e^z k^{\alpha} h^{-\alpha} n^{-\alpha-1} < 0 \tag{20}$$

$$-\gamma\frac{\partial w}{\partial n} \quad = \quad -\frac{\mu\gamma\alpha(1-\alpha)}{1-\mu\gamma\alpha}e^z k^{\alpha} h^{-\alpha} n^{-\alpha-1} > 0 \tag{21}$$

Using Equation (21), we can rewrite two important conditions (17) and (18) as follows:

$$wh = \mu\left(\frac{1-\alpha}{1-\mu\gamma\alpha}e^z k^{\alpha} h^{1-\alpha} n^{-\alpha} + \frac{V}{1-N}\kappa\right) + (1-\mu)\left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b\right) \tag{22}$$

$$\frac{\widetilde{u}_l(1-h)}{u_c} \quad = \quad \frac{1-\alpha}{1-\mu\gamma\alpha}e^z k^{\alpha}(nh)^{-\alpha} \tag{23}$$

[17] Following Andolfatto (1996) and Cheron and Langot (2004), it is assumed that a weight of each worker is small so that F_{nh} is taken as given by both the worker and the firm during the wage bargaining.

The marginal value of an additional employee to the firm can be rewritten as follows:

$$J^m = \frac{1-\alpha}{1-\mu\gamma\alpha} z k^\alpha h^{1-\alpha} n^{-\alpha} - wh + (1-\chi) E\left[\widetilde{\beta} J^{m'}\right] \tag{24}$$

Stochastic bargaining power of existing workers (γ) shows up in the equations for both intensive and extensive margins in Equation (23) and (24). This implies that bargaining shocks possibly increase the volatility of both margins. If $\gamma = 0$, we have similar conditions as in literature which uses standard bargaining protocol.

$$wh = \mu\left((1-\alpha) e^z k^\alpha h^{1-\alpha} n^{-\alpha} + \frac{V}{1-N}\kappa\right) + (1-\mu)\left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b\right) \tag{25}$$

$$\frac{\widetilde{u}_l(1-h)}{u_c} = (1-\alpha) e^z k^\alpha (nh)^{-\alpha} \tag{26}$$

$$J^m = (1-\alpha) z k^\alpha h^{1-\alpha} n^{-\alpha} - wh + (1-\chi) E\left[\widetilde{\beta} J^{m'}\right] \tag{27}$$

If $\gamma = 1$, the the conditions become similar to ones in Krause and Lubik (2013).[18]

$$wh = \mu\left(\frac{1-\alpha}{1-\mu\alpha} e^z k^\alpha h^{1-\alpha} n^{-\alpha} + \frac{V}{1-N}\kappa\right) + (1-\mu)\left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b\right) \tag{28}$$

$$\frac{\widetilde{u}_l(1-h)}{u_c} = \frac{1-\alpha}{1-\mu\alpha} e^z k^\alpha (nh)^{-\alpha} \tag{29}$$

$$J^m = \frac{1-\alpha}{1-\mu\alpha} z k^\alpha h^{1-\alpha} n^{-\alpha} - wh + (1-\chi) E\left[\widetilde{\beta} J^{m'}\right] \tag{30}$$

2.5 Government

The government simply raises revenue in order to pay out unemployment benefits b to unemployed members within the household. Therefore, the government budget constraint is

$$T(S) = (1-n) b \tag{31}$$

2.6 Equilibrium

A *recursive equilibrium* is a set of functions; the household's value function $\Omega(S, s_H)$, the household's policy functions $c(S, s_H), a'(S, s_H)$, the firm's value function $J(S, s_F)$, the

[18]Since Krause and Lubik (2013) does not have intensive margins, they do not have Equation (29).

firm's policy functions $v(S, s_f), k(S, s_f)$, aggregate prices $r(S), \widetilde{\beta}(S, S')$, taxes $T(S)$, dividends $\Pi(S)$, the law of motion for aggregate state variables $G(S)$ such that

(1) Household's policy functions solve the household's problem

(2) Firm's policy functions solve the firm's problem

(3) $\widetilde{\beta}(S, S') = \beta u(c(S'))/u_c(c(S))$

(4) Wages and hours per worker $(w(S), h(S))$ are the solution to the bargaining problem

(5) Asset market and goods market clear

(6) The government budget constraint is balanced

(7) The law of motion $G(S)$ is consistent with individual decisions

3 Calibration

First, I use the following aggregate production function and matching function.

$$F(z, k, nh) = e^z k (nh)^{1-\alpha} \tag{32}$$
$$M = \omega V^{\psi} (1 - N)^{1-\psi} \tag{33}$$

where $\alpha \in (0,1), \psi \in (0,1)$. I specify the household's utility function as follows

$$u(c) = log(c) \tag{34}$$
$$\widetilde{u}(1-h) = \phi_e \frac{(1-h)^{1-\eta}}{1-\eta} \tag{35}$$

Including the parameters in the functions defined above, I have 19 parameters to be calibrated. Parameters can be categorized into three groups based on the way to calibrate them. The first set of parameters are predetermined parameters outside the model. The second set of parameters are parameters for shock processes, which will be estimated from constructed shock processes from the U.S. data. The last group of parameters is parameters to be determined in the model by using the steady state conditions and relevant targets.

Parameters	Description	Value	Source
β	Discount factor	0.99	Annual rate of return 4%
χ	Separation rate	0.15	Andolfatto (1996)
δ	Depreciation rate	0.0102	See text and Data Appendix
α	Cobb-Douglas parameter for capital	0.36	Andolfatto (1996)
ψ	Coefficient for vacancies in matching function	0.60	Andolfatto (1996)

Table 1: Predetermined parameters

3.1 Predetermined parameters

I basically follow Andolfatto (1996) for the discount factor $\beta = 0.99$, the separation rate $\chi = 0.15$[19], the Cobb Douglas parameter for capital $\alpha = 0.36$, and the coefficient for vacancies in the matching function $\psi = 0.60$. Note that since the labor market is not competitive in this paper, I cannot use labor share data to calibrate α. I set the quarterly depreciation rate of capital stocks (δ) to be 0.0102, which is calculated when quarterly series of capital stocks are constructed by the perpetual inventory method as explained in the Data Appendix. Table 1 summarizes predetermined parameters.

3.2 Parameters for shock processes

Productivity shocks can be constructed as a series of the measured Solow residual. From the aggregate production function, we have

$$\hat{z}_t = \hat{y}_t - \alpha\hat{k}_t - (1-\alpha)\,\hat{n}_t - (1-\alpha)\,\hat{h}_t \tag{36}$$

where hats denote log-deviations from a linear trend for each variable over the period 1964:Q1-2018:Q4. The mean value of the productivity shocks ($\bar{z}$) is normalized to one. All variables are quarterly and seasonally adjusted. Nominal variables are converted to real variables using the GDP deflator. y_t is constructed using the real GDP in the NIPA and k_t is constructed by the perpetual inventory method given $\delta = 0.0102$. Employment in the CPS and average weekly

[19] When I set the separation rate to 10% instead of 15%, the relative volatility of total hours was reduced slightly in both the baseline model and the Andolfatto model. Even in this case, however, labor market volatility in the baseline model was greater than that in the Andofatto model. Therefore, the calibration for the separation rate has not changed the main message of this paper.

hours (production and nonsupervisory employees, total private) in the CES are used for n_t and h_t, respectively.

For bargaining shocks, we can use the solution to the first order differential equation (Equation (20)) that we solved before. By multiplying both sides by $\frac{n}{w}$, the labor share term appears in the right-hand side of Equation (38).

$$\frac{\partial w}{\partial n} = -\frac{\mu\alpha(1-\alpha)}{1-\mu\gamma\alpha} z k^{\alpha} h^{-\alpha} n^{-\alpha-1} \tag{37}$$

$$\frac{\partial w}{\partial n}\frac{n}{w} = -\frac{\mu\alpha(1-\alpha)}{1-\mu\gamma\alpha}\frac{y}{nhw} = -\frac{\mu\alpha(1-\alpha)}{1-\mu\gamma\alpha}\frac{1}{labor\,share} \tag{38}$$

The left-hand side of Equation (38) can be define as the elasticity ($\epsilon_{w,n} \equiv \frac{\partial w}{\partial n}\frac{n}{w}$)[20] that measures how much wages for existing workers are reduced by hiring more workers.

$$\epsilon_{w,n} = -\frac{\mu\alpha(1-\alpha)}{1-\mu\gamma\alpha}\frac{y}{nhw} = -\frac{\mu\alpha(1-\alpha)}{1-\mu\gamma\alpha}\frac{1}{labor\,share} \tag{39}$$

$$labor\,share = \frac{\mu\alpha(1-\alpha)}{1-\mu\gamma\alpha}\frac{1}{(-\epsilon_{w,n})} \tag{40}$$

In this paper, the elasticity ($\epsilon_{w,n}$) is assumed to be stable around its steady-state value ($\overline{\epsilon_{w,n}} \equiv \overline{\frac{\partial w}{\partial n}\frac{n}{w}} = -\frac{\mu\alpha(1-\alpha)}{1-\mu\bar{\gamma}\alpha}\frac{1}{\overline{labor\,share}}$) and this assumption will be turned out to be reasonable in the quantitative analysis section. From Equation (40), series of stochastic bargaining power of existing workers (γ_t) can be constructed given series of the labor share data from US.

$$(labor\,share)_t = \frac{\mu\alpha(1-\alpha)}{1-\mu\gamma_t\alpha}\frac{1}{(-\overline{\epsilon_{w,n}})} \tag{41}$$

$$\gamma_t = \frac{1}{\mu\alpha} - \frac{(1-\alpha)}{(labor\,share)_t(-\overline{\epsilon_{w,n}})} \tag{42}$$

The series of labor share are constructed from the U.S. data. Labor share is defined as $1 - \frac{\text{capital income}}{\text{GNP}}$ following Ríos-Rull and Santaeulalia-Llopis (2010). The capital income consists of unambiguous capital income and ambiguous capital income. The ambiguous capital income is computed under the assumption that the proportion of unambiguous capital income to unambiguous income is the same as the proportion of ambiguous capital income to ambiguous income. The more details on the construction of laber share can be found in the Data

[20]Note that $\epsilon_{w,n} < 0$ from Equation (38).

Parameters	**Value**	**Remarks**
ρ_{zz}	0.9578	P-value = 0.000
$\rho_{\gamma z}$	0.0016	P-value = 0.505
$\rho_{z\gamma}$	0.3192	P-value = 0.049
$\rho_{\gamma\gamma}$	0.9398	P-value = 0.000
σ_z	0.0061	-
σ_γ	0.0508	-
$\sigma_{z\gamma}$	-0.0001	$\rho(\varepsilon_z, \varepsilon_\gamma)$ =-0.2842

Table 2: Shock processes

Appendix. Given the constructed labor share, real wages are defined as $\frac{\text{labor share}\times\text{output}}{\text{total hours}}$.

Given the signs of parameters and the elasticity ($\overline{\epsilon_{w,n}} < 0$), there exists the positive relationship between stochastic bargaining power (γ_t) and labor share. This implies that the higher labor share is related to the higher bargaining powers of existing workers.

$$\frac{\partial\,(labor\ share)}{\partial\gamma} = \frac{(\mu\alpha)^2\,(1-\alpha)}{(1-\mu\gamma\alpha)^2\,(-\overline{\epsilon_{w,n}})} > 0 \tag{43}$$

Based on several information criteria such as SBIC and HQIC, I specify VAR(1) system for detrended shock series $(\hat{z}, \hat{\gamma})$ to estimate shock processes.

$$\begin{pmatrix} \hat{z}' \\ \hat{\gamma}' \end{pmatrix} = \begin{pmatrix} \rho_{zz} & \rho_{\gamma z} \\ \rho_{z\gamma} & \rho_{\gamma\gamma} \end{pmatrix} \begin{pmatrix} \hat{z} \\ \hat{\gamma} \end{pmatrix} + \begin{pmatrix} \varepsilon_z' \\ \varepsilon_\gamma' \end{pmatrix} \tag{44}$$

$$\begin{pmatrix} \varepsilon_z \\ \varepsilon_\gamma \end{pmatrix} \sim N\left(0, \begin{pmatrix} \sigma_{zz}^2 & \sigma_{z\gamma} \\ \sigma_{z\gamma} & \sigma_{\gamma\gamma}^2 \end{pmatrix}\right) \tag{45}$$

Table 2 summarizes parameters estimated using VAR(1) system above. All coefficient parameters except for $\rho_{\gamma z}$ are statistically significant at 5% level of significance. Note that we have $\rho_{z\gamma} = 0.3192$, which means that today's productivity shocks increase tomorrow's bargaining powers of existing workers. This is the key mechanism that the inclusion of the stochastic bargaining makes total hours, employment and hours per workers more volatile in addition to productivity shocks.

Target	Value	Source
Frisch elasticity of hours for those employed	0.50	Andolfatto (1996)
Steady-state employment to population ratio	0.60	Data (1964:Q1-2018:Q4)
Steady-state hours per worker	0.35	Data (1964:Q1-2018:Q4)
Steady-state job-filling rate	0.90	Andolfatto (1996)
Vacancy expenditure to output ratio	0.0227	Silva & Toledo (2009)
Replacement ratio	0.40	Shimer (2005)
$\mu = \bar{\gamma}$	-	Jointly determined in the model

Table 3: Targets

Parameters	Description	Baseline
η	Curvature parameter for leisure	3.7172
ϕ_e	Scale parameter for leisure	0.8753
κ	Cost of posting vacancies	0.2398
ω	Matching efficiency	0.5178
b	Unemployment Benefits	0.4951
μ	Bargaining power of a marginal worker	0.5742
$\overline{\gamma}$	Bargaining power of existing workers	0.5742

Table 4: Parameters determined using targets

3.3 Parameters determined using targets

I choose the remaining 7 parameters using equilibrium conditions in the steady state, targets from the literature, and the U.S. data over 1964:Q1-2018:Q4. The targets used in the calibration are summarized in Table 3. First, I set Frisch elasticity of hours for workers to 0.50, the steady state job-filling rate to 0.90 as in Andolfatto (1996), which is common across the literature. According to Silva and Toledo (2009), the average cost of time spent hiring one worker is approximately 3.6%-4.3% of total labor costs. I take the target the mid point of those range, 3.9%, which gives vacancy expenditure to output ratio $\frac{\kappa v}{y} = 0.0227$.[21] I use 40 percent as the value of unemployment benefits following Shimer (2005). In Shimer (2005), this value implicitly includes the value of leisure, but in this paper I explicitly consider the leisure in the utility function, and therefore unemployment benefits (b) are purely unemployment benefits as in Nakajima (2012).

Parameters determined using these targets are listed in Table 4. The curvature parameter

[21] The average cost of time spent hiring one worker in the model can be defined as $\frac{\kappa v}{\Phi w n h}$. Given the job-filling rate ($\Phi = 0.90$) and labor share (0.6458), we have $\frac{\kappa v}{y} = 0.0227$.

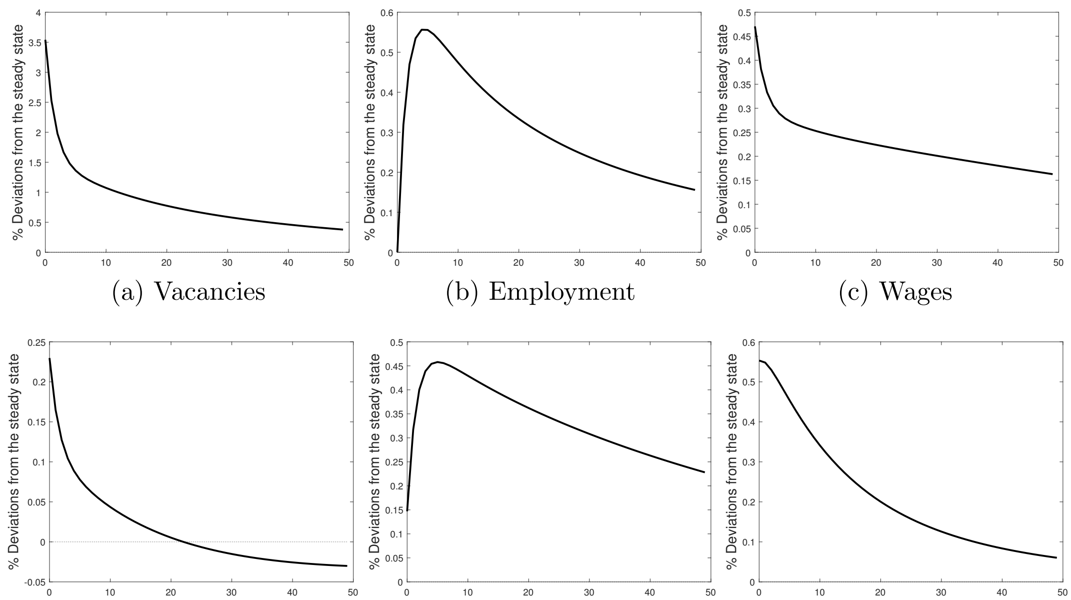
of leisure (η) is set to 3.7172 by using the Frisch elasticity of hours for those employed and the steady-state hours per worker. The cost of posting vacancies (κ) is calibrated as 0.2398 by matching the vacancy expenditure to output ratio in the steady-state of the model with that in data. The matching efficiency (ω) is computed as 0.5178 by using the coefficient for vacancies in matching function (ψ) and the steady-state values for job-filling rate, employment to population ratio, and vacancies. The remaining four parameters are jointly determined by using four equilibrium conditions in the steady state. In this process the remaining targets and already determined parameters are used. Note that the mean value of the bargaining power of existing workers ($\overline{\gamma}$) is set as 0.5742, which is the same as the bargaining power of a marginal worker (μ) calibrated in the model as assumed in Table 3.[22] As discussed in the robustness section later, lower values of $\overline{\gamma}$ generates more volatile labor market variables. However, $\overline{\gamma} = \mu = 0.5742$ gives almost the least volatility among $\overline{\gamma} \in (0, 1)$ in the baseline model. In this regard, the choice of $\overline{\gamma} = 0.5742$ seems parsimonious. Also, note that calibrated value for the bargaining power of a marginal worker (μ) is 0.5742, which guarantees quantitative results of the baseline model are not a direct result from a low value of μ as noted in Hagedorn and Manovskii (2008). Given the calibrated parameters and the mean value of labor share (0.6458), the elasticity ($\overline{\epsilon_{w,n}}$) is calculated as -0.2325, which implies that when a firm hires more new workers by 1%, wages for existing workers decrease by 0.23%.

4 Quantitative analysis

4.1 Impulse response functions to positive bargaining shocks

Figure 1 shows the impulse response of key labor market variables to the positive one standard deviation bargaining shock. When positive bargaining shocks hit the economy, the bargaining power of existing workers instantly increases. Since bargaining powers of existing workers are higher than before, a firm has more incentive to hire marginal workers by offering

[22] The mean value of the bargaining power of existing workers ($\overline{\gamma}$) and the bargaining power of a new worker (μ) are jointly determined in the model by satisfying $\overline{\gamma} = \mu$ and other equilibrium conditions in the steady state.



(a) Vacancies (b) Employment (c) Wages

(d) Hours per worker (e) Output (f) Labor share

Figure 1: IRFs to the positive one standard deviation bargaining shock

higher wages and hours per worker to forgo the higher cost associated with the failure to hire marginal workers. Therefore, the firm instantly posts more vacancies, and employment increases one period later due to the nature of search frictions. Since the firm hires more workers by offering higher hours per worker, total hours increase resulting in higher outputs in the equilibrium. Higher employment, hours per worker, and wages results in an increase in labor share by offsetting an increase in outputs.

4.2 Business cycle moments

Table 5 summarizes quantitative results of the baseline model. I compare the baseline model to the Andolfatto model to see what gains and what shortcomings the inclusion of bargaining shocks gives. Again, all data are in log and HP filtered. First of all, the baseline model generates a high (relative) volatility of employment (0.74) which almost close to the actual U.S. data (0.68). This is a remarkable success and the main contribution in this paper. The pro-cyclical bargaining power of existing workers enhances the volatility of labor market variables despite more flexible wages relative to the standard business cycle model with search

	$\sigma_x\%\left(\frac{\sigma_x}{\sigma_{Output}}\right)$			$\rho\left(x, Output\right)$			$\rho\left(x_t, x_{t-1}\right)$		
Variable (x)	Data	Baseline	Andolfatto	Data	Baseline	Andolfatto	Data	Baseline	Andolfatto
Output	1.45 (1.00)	1.11 (1.00)	1.14 (1.00)	1.00	1.00	1.00	0.87	0.82	0.82
Total Hours	1.28 **(0.88)**	0.91 **(0.82)**	0.64 **(0.56)**	0.87	0.75	0.92	0.90	0.91	0.91
Employment	0.98 **(0.68)**	0.82 **(0.74)**	0.60 **(0.53)**	0.81	0.71	0.78	0.92	0.89	0.89
Hours per Worker	0.42 **(0.29)**	0.26 **(0.24)**	0.18 **(0.16)**	0.74	0.38	0.68	0.79	0.54	0.54
Wages	0.91 **(0.63)**	0.66 **(0.59)**	0.52 **(0.45)**	0.27	0.76	0.93	0.74	0.61	0.61
Labor Productivity	0.72 (0.50)	0.74 (0.67)	0.61 (0.53)	0.47	0.58	0.91	0.65	0.68	0.59
Labor Share	0.76 **(0.53)**	0.82 **(0.74)**	0.10 **(0.09)**	**-0.12**	**0.09**	**-0.73**	**0.76**	**0.73**	**0.47**
Vacancies	13.30 (9.18)	4.38 (3.95)	3.21 (2.81)	0.90	0.60	0.81	0.92	0.52	0.52

1) All data are in logs and filtered using the HP filter with a smoothing parameter of 1600.
2) In the Andolfatto model, the standard bargaining protocol is used.
3) Data for vacancies are only available up to 2016:Q4.

Table 5: Business cycle moments in data and models over 1964:Q1-2018:Q4

frictions. The relative volatility of wages in the model (0.59) is similar to that in the data (0.63).[23] Despite the more flexible wages, the relative volatility of employment, hours per worker, and total hours are higher than those in the Andolfatto model.

Since employment is very volatile, total hours is much volatile than the Andolfatto model. Hours per worker and vacancies are slightly more volatile than Andolfatto, but the differences are small. The moments for labor share are similar to the actual U.S. data. This result might be a direct result of the identification strategy for bargaining shocks from labor share data. However, the moments for labor share in the model, along with the overshooting property I will discuss shortly, justify the assumption for the identification of bargaining shocks; the elasticity ($\epsilon_{w,n} \equiv \frac{\partial w}{\partial n}\frac{n}{w}$) is stable around the steady state.

The main mechanism generates more volatile labor market variables is that the impact of productivity shocks is amplified by changes in bargaining powers of existing workers in addition to the impact of each shock. Recall that the estimated parameter for $\rho_{z\gamma}$ is 0.3192.

[23] Although the wages in the baseline model are less pro-cyclical than those in the Andolfatto model, the wages in the baseline model are too pro-cyclical compared with those in the data. As noted in Cheron and Langot (2004), a strong pro-cyclical real wage is a common problem in business cycle models with search frictions and wage bargaining. When conventional specifications for preferences (i.e., utility functions) are used, outside options for workers behave in a pro-cyclical manner, leading to upward pressure on wages during expansions. Cheron and Langot (2004) demonstrate that when a non-separable preference between consumption and leisure is used, the pro-cyclicality of wages in a model is reduced. The problem of incorporating a non-separable preference into my model is that wage rigidity (i.e., less volatile wages) is also included in the model. Given that the objective of this study is to increase the volatility of labor market quantities without adding wage rigidity, I have not used a non-separable preference to decrease the pro-cyclicality of real wages.

This means that as the productivity shocks today positively affect the bargaining powers of existing workers tomorrow. As the bargaining power of existing workers increases, the firm will have more incentive to hire marginal workers by offering higher wages and hours per worker to forgo the higher cost associated with the failure to hire marginal workers. This dynamic interaction between productivity shocks and bargaining shocks amplifies the volatility of labor market variables.

I now consider shortcomings of the baseline model relative to Andolfatto model. The baseline model generates the higher volatility of labor productivity, weak pro-cyclicality of total hours, employment and hours per worker. Despite these shortcomings, the baseline model performs better than Andolfatto model in general. This result mainly comes from the time-varying bargaining power of existing workers and hence time-varying firms' incentive to hire workers.

4.3 Implications for labor share dynamics

Ríos-Rull and Santaeulalia-Llopis (2010) first document the overshooting property of labor share. They show that labor share overshoots in response to productivity shocks, and the dynamic overshooting response of labor share drastically dampens the role of productivity shocks on labor markets due to huge wealth effects. Figure 2 shows the overshooting of labor share in the baseline model.[24] To represent the overshooting property of labor share, productivity and bargaining shocks should be correlated. The model no longer generates the overshooting of labor share when bargaining shocks are abstracted from or productivity and bargaining shocks have zero correlation ($\rho_{z\gamma} = 0$) as shown in Figure 2. Therefore, the overshooting property of labor share is purely driven by the features of exogenous shocks similar to that in Ríos-Rull and Santaeulalia-Llopis (2010).

More importantly, the baseline model generates the overshooting property of labor share, but the effect of productivity shocks is still significant on labor markets in contrast to the

[24]The reason the model with bargaining shocks features the overshooting of labor share may be a direct result of the identification strategy of bargaining shocks. Again, the fact that labor share overshoots in the baseline model, along with other moments for labor share are very similar to those in actual data, justifies the assumption I pose to identify bargaining shocks; the elasticity ($\epsilon_{w,n}$) does not move much around the steady state.

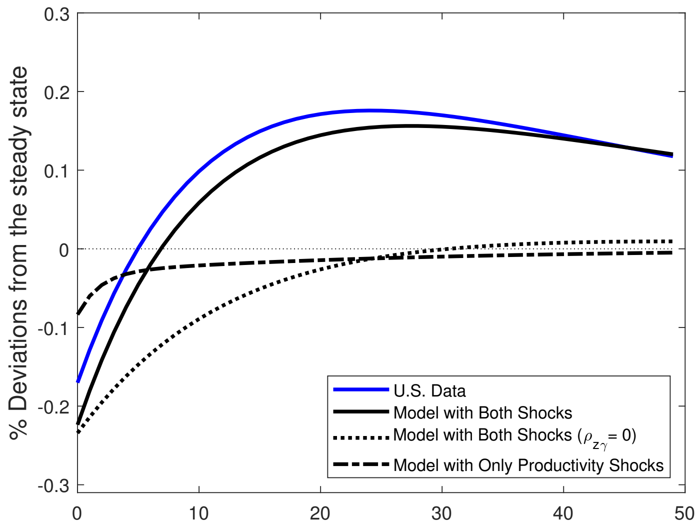


Figure 2: Impulse response function of labor share to productivity innovation

prediction of Ríos-Rull and Santaeulalia-Llopis (2010) in which the effect of productivity shocks is dampened when labor share overshoots because of huge wealth effects from the overshooting property. In contrast to their model, the baseline model has a search and matching framework, and the nature of this framework weakens wealth effects resulting from the overshooting of labor share. On top of these differences, more incentive for firms to hire workers due to bargaining shocks offset the huge reduction of total hours in booms. In response to positive productivity shocks output instantly increases, but employment does not increase because of search frictions, which cause an instant drop in labor share. As the productivity shocks today positively affect the bargaining shocks tomorrow, $\rho_{z\gamma} = 0.3192$, and as the bargaining power of existing workers increases, the firm will have more incentive to hire marginal workers by offering higher wages and hours per worker. Consequently, employment, wages, and hours per worker will increase by offsetting an increase in outputs. This increase explains the overshooting of labor share in response to positive productivity shocks.

	$\sigma_x\%\left(\frac{\sigma_x}{\sigma_{Output}}\right)$			$\rho\,(x, Output)$			$\rho\,(x_t, x_{t-1})$		
Variable (x)	Both shocks	Only z	Only γ	Both shocks	Only z	Only γ	Both shocks	Only z	Only γ
Output	1.11 (1.00)	1.14 (1.00)	0.55 (1.00)	1.00	1.00	1.00	0.82	0.82	0.91
Total Hours	0.91 **(0.82)**	0.64 **(0.56)**	0.87 **(1.58)**	0.75	0.92	1.00	0.91	0.91	0.91
Employment	0.82 **(0.74)**	0.60 **(0.53)**	0.75 **(1.36)**	0.71	0.78	0.96	0.89	0.89	0.88
Hours per Worker	0.26 **(0.24)**	0.18 **(0.15)**	0.28 **(0.51)**	0.38	0.68	0.53	0.54	0.53	0.55
Wages	0.66 **(0.59)**	0.53 **(0.46)**	0.54 **(0.98)**	0.76	0.93	0.66	0.61	0.61	0.58
Labor Productivity	0.74 (0.67)	0.61 (0.53)	0.32 (0.58)	0.58	0.91	-0.98	0.68	0.59	0.91
Labor Share	0.82 **(0.74)**	0.09 **(0.08)**	0.80 **(1.46)**	**0.09**	**-0.72**	**0.84**	**0.73**	**0.47**	**0.75**
Vacancies	4.38 (3.95)	3.23 (2.82)	4.03 (7.33)	0.60	0.81	0.52	0.52	0.51	0.51

Table 6: Business cycle moments in models with different shocks

4.4 The Role of productivity shocks and bargaining shocks

Now I consider how productivity shocks and bargaining shocks differently affect the model predictions.[25] Table 6 shows quantitative results of models with only productivity or bargaining shocks. When the economy has only productivity shocks, the model predictions are almost the same as the Andolfatto model. Comparing to the baseline model which has both shocks, the volatility of employment, labor share, and vacancies is dampened. Correlations between labor market variables and outputs increase in general except for labor share. Auto-correlations are almost the same as the baseline case except for labor productivity and labor share.

When the economy has only bargaining shocks, the volatility of outputs significantly drops, which means bargaining shocks cannot be the main driving source of output fluctuations. On the other hand, the volatility of total hours, employment, and hours per worker remarkably increases, which is far beyond the volatility in the baseline model. Also, total hours, employment, and labor shares are strongly pro-cyclical. However, auto-correlations are almost the same as the baseline case except for output and labor productivity.

Table 7 shows the variance decomposition. Bargaining shocks have a substantial impact on the volatility of total hours, employment, hours per worker, wages, labor share, and vacancies. While bargaining shocks play a remarkable role in the labor markets, productivity shocks seem

[25] Given the series of labor share, shock processes do not affect the calibration of other parameters in the model. Therefore, the same parameters as the baseline model except for shock processes are used in each experiment.

Variable	Productivity shocks (z)	Bargaining shocks (γ)
Output	78.21	21.79
Total Hours	24.24	75.76
Employment	29.62	70.38
Hours per Worker	7.93	92.07
Wages	37.54	62.46
Labor Productivity	85.01	14.99
Labor Share	14.98	85.02
Vacancies	26.92	73.08

Table 7: Variance decomposition (in percent)

to be still the main driving force of business cycles given that productivity shocks account for about 78% of the output fluctuations. This result is also consistent with the finding in moments in Table 6.

5 Robustness

5.1 Stochastic bargaining power of a marginal worker (μ_t)

Now I assume the bargaining power of a marginal worker varies stochastically while the bargaining power of existing workers is fixed at $\gamma = \overline{\gamma} = \overline{\mu}$. Again, series of μ_t can be identified by using the solution to the first order differential equation, and series of the labor share data from US.

$$
\begin{aligned}
(labor\, share)_t &= \frac{\mu_t \alpha (1-\alpha)}{\mu_t \bar{\gamma} \alpha} \frac{1}{(-\overline{\epsilon_{w,n}})} && (46) \\
\mu_t &= \frac{1}{\bar{\gamma}\alpha + \frac{\alpha(1-\alpha)}{(labor\, share)_t (-\overline{\epsilon_{w,n}})}} && (47)
\end{aligned}
$$

where $\epsilon_{w,n} \equiv \frac{\partial w}{\partial n}\frac{n}{w}$. Again, it is assumed that the elasticity ($\epsilon_{w,n}$) is stable around the steady-state value ($\overline{\epsilon_{w,n}} \equiv \overline{\frac{\partial w}{\partial n}\frac{n}{w}} = -\frac{\bar{\mu}\alpha(1-\alpha)}{1-\bar{\mu}\bar{\gamma}\alpha}\frac{1}{\overline{labor\, share}}$). Table 8 shows the comparison of business cycle moments. Stochastic bargaining power of a marginal worker cannot quantitatively improve the Andolfatto model, even moments for labor share which is used for identifying shock series (μ_t).[26]

[26]This result does not change with different values of $\bar{\gamma}$.

	$\sigma_x\%\left(\frac{\sigma_x}{\sigma_{Output}}\right)$		$\rho\left(x, Output\right)$		$\rho\left(x_t, x_{t-1}\right)$	
Variable (x)	Shock on μ	Andolfatto	Shock on μ	Andolfatto	Shock on μ	Andolfatto
Output	1.16 (1.00)	1.14 (1.00)	1.00	1.00	0.83	0.82
Total Hours	0.69 (0.59)	0.64 (0.56)	0.91	0.92	0.91	0.91
Employment	0.67 (0.58)	0.60 (0.53)	0.79	0.78	0.88	0.89
Hours per Worker	0.17 (0.15)	0.18 (0.16)	0.60	0.68	0.51	0.54
Wages	0.49 (0.42)	0.52 (0.45)	0.89	0.93	0.60	0.61
Labor Productivity	0.60 (0.52)	0.61 (0.53)	0.88	0.91	0.57	0.59
Labor Share	0.15 (0.13)	0.10 (0.09)	-0.60	-0.73	0.56	0.47
Vacancies	3.55 (3.07)	3.21 (2.81)	0.77	0.81	0.51	0.52

Table 8: Business cycle moments in model: shocks on μ

	$\sigma_x\%\left(\frac{\sigma_x}{\sigma_{Output}}\right)$			$\rho\left(x, Output\right)$			$\rho\left(x_t, x_{t-1}\right)$		
Variable (x)	$\overline{\gamma}=0.4$	$\overline{\gamma}=0.5742$	$\overline{\gamma}=0.8$	$\overline{\gamma}=0.4$	$\overline{\gamma}=0.5742$	$\overline{\gamma}=0.8$	$\overline{\gamma}=0.4$	$\overline{\gamma}=0.5742$	$\overline{\gamma}=0.8$
Output	1.23 (1.00)	1.11 (1.00)	1.09 (1.00)	1.00	1.00	1.00	0.84	0.82	0.82
Total Hours	1.27 (1.03)	0.91 (0.82)	0.85 (077)	0.78	0.75	0.75	0.91	0.91	0.91
Employment	1.11 (0.91)	0.82 (0.74)	0.77 (0.70)	0.76	0.71	0.70	0.88	0.89	0.89
Hours per Worker	0.40 (0.33)	0.26 (0.24)	0.24 (0.22)	0.38	0.38	0.39	0.54	0.54	0.53
Wages	0.86 (0.70)	0.66 (0.59)	0.62 (0.57)	0.69	0.76	0.79	0.59	0.61	0.61
Labor Productivity	0.82 (0.67)	0.74 (0.67)	0.72 (0.66)	0.28	0.58	0.63	0.72	0.68	0.67
Labor Share	1.20 (0.98)	0.82 (0.74)	0.74 (0.67)	0.30	0.09	0.04	0.73	0.73	0.73
Vacancies	5.98 (4.88)	4.38 (3.95)	4.08 (3.73)	0.53	0.60	0.63	0.51	0.52	0.52

Table 9: Business cycle moments in the model with different values for $\overline{\gamma}$

5.2 Calibration of $\overline{\gamma}$

I now simulate the baseline model with different values for $\overline{\gamma}$; 0.4[27] (an example of low values), 0.5742 (a middle value and the calibrated value for the baseline model such that $\mu = \overline{\gamma}$), and 0.8[28] (an example of high values). Table 9 shows business cycle moments for each case. If I set $\overline{\gamma} = 0.4$, then volatility of employment and hours per workers significantly increases than the baseline calibration case, $\overline{\gamma} = 0.5742$. However, if I set $\overline{\gamma} = 0.8$, then moments are almost the same as those of the baseline calibration case, $\overline{\gamma} = 0.5742$. Mechanically, low values of $\overline{\gamma}$ increase the volatility of total hours, employment, and hours per worker. Given that there is no clear way to pin down $\overline{\gamma}$, the choice of $\overline{\gamma} = 0.5697$ in the baseline model is parsimonious in the sense that $\overline{\gamma} = 0.5742$ yields the almost least volatility of labor market variables among

[27]In this case, the calibrated value for μ is 0.5620.
[28]In this case, the calibrated value for μ is 0.5915.

	$\sigma_x\%\left(\frac{\sigma_x}{\sigma_{Output}}\right)$			$\rho(x, Output)$			$\rho(x_t, x_{t-1})$		
Variable (x)	Data	Baseline	Andolfatto	Data	Baseline	Andolfatto	Data	Baseline	Andolfatto
Output	1.45 (1.00)	1.01 (1.00)	1.14 (1.00)	1.00	1.00	1.00	0.87	0.82	0.82
Total Hours	1.28 (0.88)	0.87 (0.86)	0.64 (0.56)	0.87	0.69	0.92	0.90	0.91	0.91
Employment	0.98 (0.68)	0.77 (0.76)	0.60 (0.53)	0.81	0.69	0.78	0.92	0.89	0.89
Hours per Worker	0.42 (0.29)	0.28 (0.27)	0.18 (0.15)	0.74	0.25	0.68	0.79	0.52	0.53
Wages	0.81 (0.56)	0.64 (0.63)	0.52 (0.46)	0.36	0.72	0.93	0.72	0.61	0.61
Labor Productivity	0.72 (0.50)	0.75 (0.74)	0.61 (0.53)	0.47	0.55	0.91	0.65	0.65	0.59
Labor Share	0.79 (0.55)	0.90 (0.89)	0.09 (0.08)	-0.06	0.05	-0.72	0.56	0.67	0.47
Vacancies	13.30 (9.18)	4.06 (4.01)	3.22 (2.82)	0.90	0.61	0.81	0.92	0.52	0.52

Table 10: Business cycle moments in model: labor share $=\frac{compensation\, of\, employees}{GNP - proprietors'\, income}$

	$\sigma_x\%\left(\frac{\sigma_x}{\sigma_{Output}}\right)$			$\rho(x, Output)$			$\rho(x_t, x_{t-1})$		
Variable (x)	Data	Baseline	Andolfatto	Data	Baseline	Andolfatto	Data	Baseline	Andolfatto
Output	1.45 (1.00)	0.99 (1.00)	1.14 (1.00)	1.00	1.00	1.00	0.87	0.81	0.82
Total Hours	1.28 (0.88)	0.88 (0.88)	0.64 (0.56)	0.87	0.63	0.92	0.90	0.91	0.91
Employment	0.98 (0.68)	0.76 (0.77)	0.60 (0.53)	0.81	0.65	0.78	0.92	0.89	0.89
Hours per Worker	0.42 (0.29)	0.28 (0.28)	0.18 (0.15)	0.74	0.22	0.68	0.79	0.54	0.53
Wages	0.77 (0.53)	0.62 (0.62)	0.53 (0.46)	0.23	0.69	0.93	0.71	0.61	0.61
Labor Productivity	0.72 (0.50)	0.81 (0.81)	0.61 (0.53)	0.47	0.54	0.91	0.65	0.70	0.59
Labor Share	0.81 (0.56)	0.95 (0.96)	0.08 (0.07)	-0.20	-0.02	-0.72	0.61	0.71	0.47
Vacancies	13.30 (9.18)	4.05 (4.08)	3.23 (2.83)	0.90	0.54	0.81	0.92	0.52	0.51

Table 11: Business cycle moments in model: labor share $=\frac{compensation\, of\, employees}{GNP}$

$\overline{\gamma} \in (0, 1)$.

5.3 Definition of labor share

In the baseline model, I use the same definition of labor share as in Ríos-Rull and Santaeulalia-Llopis (2010). In order to show that the main results of this paper is robust to the definition of labor share, the same analysis is conducted with the two different definitions of labor share: $\frac{compensation\, of\, employees}{GNP - proprietors'\, income}$ and $\frac{compensation\, of\, employees}{GNP}$. For each definition of labor share, the shock processes are re-estimated and the parameters determined in the model are re-calibrated due to different value of steady-state labor share.[29] Table 10 and Table 11 show that the main results hold regardless of the definition of labor share although the volatility

[29]The elasticity $(\epsilon_{w,n})$ in the model using two different definitions of labor share is -0.2710 and -0.3061, respectively.

of output slightly decreases.

6 Implications for the trend decline in labor share

Recently, various studies have been conducted on the trend decline in labor share in the U.S. and other countries. Elsby et al. (2013) show that the offshoring of the labor-intensive part of the U.S. supply chain is a major factor for the decline in U.S. labor share over the past 25 years. Karabarbounis and Neiman (2014) argue that the decline in the relative price of investment explains approximately half of the decline in global labor share given the estimate of the elasticity of substitution between capital and labor (i.e., 1.25). Autor et al. (2020) provide an alternative hypothesis for the decline in labor share that is based on the rise of superstar firms in the sense that labor share falls as superstar firms gain market share across a wide range of sectors. Lastly, Koh et al. (2020) find that the decline in labor share is fully explained by the accounting treatment of intellectual property products in the national income and product accounts.

Discussion on the relationship between the fall in union power and the declining labor share have also been conducted recently. Krueger (2018) argues that declining union power will be a potential mechanism that contributes to the decline in labor share given the secular decrease in union density. Elsby et al. (2013) and Autor et al. (2020) also acknowledge that deunionization can be an important factor in the declining labor share, although it is not a major factor. My study deals with the movement of labor share over business cycles, but it also provides implications for the trending decline in labor share in terms of the bargaining power of workers. The deunionization hypothesis is consistent with the long-run prediction of this study, although union is not explicitly modeled in this study. In this study, the trend in the bargaining power decrease of existing workers (possibly due to deunionization) leads to the trend in labor share decline as shown in Equation (41). Moreover, union power reflects the bargaining power of existing workers rather than the bargaining power of marginal workers who are outside labor unions. From this perspective, if the relationship between the bargaining power of existing workers and deunionization can be empirically measured, then the model presented in this study is expected to be used for quantifying the effect of deunionization on the secular decline in labor share.

7 Conclusion

This paper develops a business cycle search and matching model in which the volatility of employment closely matches that of the U.S. data without wage rigidity and the labor share overshoots in response to productivity shocks. Specifically, I introduce an alternative mechanism of wage negotiations and bargaining shocks of existing workers in multi-worker firms that face diminishing MPL. Due to diminishing MPL, the breakdown in the negotiation with the marginal worker negatively affects the bargaining position of the firm with existing workers since MPL is higher with one fewer workers. How much the firm internalizes this negative effect depends on the stochastic bargaining powers of existing workers which can be identified through labor share data. The calibrated model generates more volatile total hours, employment, hours per worker, and wages while labor share overshoots in response to productivity shocks as documented in Ríos-Rull and Santaeulalia-Llopis (2010). The bargaining power of existing workers affects the outside option value for firms. The pro-cyclical bargaining power of existing workers results in the counter-cyclical firm's outside option and the pro-cyclical firm's surplus. Consequently, the pro-cyclical bargaining power of existing workers leads to more flexible wages and more volatile employment.

In this paper, the bargaining power of existing workers is assumed to be exogenous. The quantitative results show that the time-varying bargaining power of existing workers is an important margin to understand the labor market fluctuations including the overshooting property of labor share. However, this paper abstracts from an endogenous mechanism for the time-varying bargaining power of existing workers. Therefore, investigating endogenous mechanisms for time-varying bargaining power of existing workers would be worthwhile for future research. In this regard, Mangin and Sedlacek (2018) suggest one direction to endogenize the time-varying bargaining power of existing workers. In booms, the competition among firms can be intensified due to higher entry rates of new firms and lower exit rates of existing firms, which reduces the bargaining power of firms over existing workers. However, there is a limitation in that their model still requires another type of shocks, investment-specific technology shocks, in addition to aggregate productivity shocks in order to generate reasonable labor market dynamics. Therefore, research on endogenous mechanisms that increase the volatility of entry and exit rates of firms without investment-specific technology shocks may be helpful for understanding time-varying bargaining powers of workers and labor market fluctuations.

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Appendix

Derivation of the equilibrium conditions

Household solves the following dynamic programming problem.

$$\Omega\left(S, s_H\right) = \max_{c,a'} u\left(c\right) + n\widetilde{u}\left(1 - h\left(S, s_H\right)\right) + \left(1-n\right)\widetilde{u}\left(1\right) + \beta E\left[\Omega\left(S', s'_H\right)\right]$$

$$s.t.$$

$$c + a' + T\left(S\right) = nw\left(S, s_H\right)h\left(S, s_H\right) + \left(1-n\right)b + \left(1 + r(S)\right)a + \Pi\left(S\right)$$

$$n' = \left(1-\chi\right)n + p\left(S\right)\left(1-n\right)$$

$$S' = G\left(S\right)$$

Let λ_H and μ_H be the Lagrange multiplier on budget constraint and law of motion for employment respectively. Then, we have the following first order conditions.

$$\begin{aligned} u_c &= \lambda_H \\ E\left[\beta\Omega'_a\right] &= \lambda_H \end{aligned}$$

From the envelope condition with respect to a, we get

$$\Omega_a = \left(1+r\right)\lambda_H$$

Taking a derivative with respect to n', we have

$$\mu_H = E\left[\beta\Omega'_n\right]$$

By combining equations above, we have the standard Euler equation.

$$E\left[\beta\frac{u'_c}{u_c}\left(1+r'\right)\right] = 1$$

Now, a firm solves the following problem.

$$\begin{aligned}
J(S,s_F) &= \max_{v,k} \Pi(S) + E\left[\widetilde{\beta}(S,S')\,J(S',s'_F)\right] \\
&= \max_{v,k} F(z,k,nh) - w(S,s_F)\,h(S,s_F)\,n - (r(S)+\delta)\,k - \kappa v + E\left[\widetilde{\beta}(S,S')\,J(S',s'_F)\right] \\
s.t. & \\
n' &= (1-\chi)\,n + q(S)\,v \\
S' &= G(S)
\end{aligned}$$

where $\widetilde{\beta}(S,S') = \beta u_c(c(S'))/u_c(c(S))$ is the stochastic discount factor and $q(S) = M/V$ is the job-filling rate.

Let μ_F be the Lagrange multipliers on law of motion of employment. Then, we have the following first order conditions for the firm.

$$\begin{aligned}
\kappa &= \mu_F q(S) \\
r+\delta &= F_k
\end{aligned}$$

From the definition of the marginal value of an additional employee to the firm $\left(J^m \equiv \frac{\partial J}{\partial n}\right)$, the following condition should hold.

$$E\left[\widetilde{\beta} J^{m'}\right] = \mu_F$$

By combining equations above, we have an equation for the rental rate and the job creation condition.

$$\begin{aligned}
r &= F_k - \delta \\
\kappa &= qE\left[\widetilde{\beta} J^{m'}\right]
\end{aligned}$$

Derivation of the marginal value of an additional employee to the firm

$$
\begin{aligned}
J^m &= \lim_{\Delta\to 0}\frac{1}{\Delta}\left[\left(F(z,k,(n+\Delta)h) - w[n+\Delta](n+\Delta)h - (r+\delta)k - \kappa v + \beta E\left[\frac{u_c^{'}}{u_c}J\left[(n+\Delta)^{'}\right]\right]\right)\right. \\
&\quad \left. -\left(F(z,k,nh) - w^e[n]nh - (r+\delta)k - \kappa v + \beta E\left[\frac{u_c^{'}}{u_c}J^B\left[n^{'}\right]\right]\right)\right] \\
&= \lim_{\Delta\to 0}\frac{F(z,k,(n+\Delta)h) - F(z,k,nh)}{\Delta} - \lim_{\Delta\to 0}\frac{w[n+\Delta](n+\Delta)h - w^e[n]nh}{\Delta} \\
&\quad + E\left[\beta\frac{u_c^{'}}{u_c}\lim_{\Delta\to 0}\frac{J[(1-\chi)(n+\Delta)] - J^B[(1-\chi)n]}{\Delta}\right] \\
&= \frac{\partial F(z,k,nh)}{\partial n} - \lim_{\Delta\to 0}\frac{w[n+\Delta](n+\Delta)h - w^e[n]nh}{\Delta} \\
&\quad + (1-\chi)E\left[\beta\frac{u_c^{'}}{u_c}\lim_{(1-\chi)\Delta\to 0}\frac{J[(1-\chi)n + (1-\chi)\Delta] - J^B[(1-\chi)n]}{(1-\chi)\Delta}\right] \\
&= F_n - \lim_{\Delta\to 0}\frac{w[n+\Delta](n+\Delta)h - w^e[n]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right]
\end{aligned}
$$

Proofs of Propositions

Proof of Proposition 1

Under $w^e[n] = w[n+\triangle]$, Equation (10) can be rewritten as

$$
\begin{aligned}
J^m &= F_n - \lim_{\Delta\to 0}\frac{w[n+\Delta](n+\Delta)h - w[n+\Delta]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right] \\
&= F_n - \lim_{\Delta\to 0}\frac{w[n+\Delta]nh + w[n+\Delta]\Delta h - w[n+\triangle]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right] \\
&= F_n - \lim_{\Delta\to 0} w[n+\Delta]h + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right] \\
&= F_n - wh + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right]
\end{aligned}
$$

Proof of Proposition 2

Under $w^e[n] = w[n]$, Equation (10) can be rewritten as

$$\begin{aligned}
J^m &= F_n - \lim_{\Delta\to 0} \frac{w[n+\Delta](n+\Delta)h - w[n]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c} J^{m'}\right] \\
&= F_n - \lim_{\Delta\to 0} \frac{w[n+\Delta]nh + w[n+\Delta]\Delta h - w[n]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c} J^{m'}\right] \\
&= F_n - \lim_{\Delta\to 0} \frac{w[n+\Delta] - w[n]}{\Delta} nh - \lim_{\Delta\to 0} w[n+\Delta]h + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c} J^{m'}\right] \\
&= F_n - \frac{\partial w[n]}{\partial n} nh - w[n]h + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c} J^{m'}\right] \\
&= F_n - w[n]h - \frac{\partial w[n]}{\partial n} nh + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c} J^{m'}\right]
\end{aligned}$$

Proof of Proposition 3

Under $w^e[n] = \gamma w[n] + (1-\gamma) w[n+\Delta]$, Equation (10) can be rewritten as

$$\begin{aligned}
J^m &= F_n - \lim_{\Delta\to 0} \frac{w[n+\Delta](n+\Delta)h - (\gamma w[n] + (1-\gamma)w[n+\Delta])nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c} J^{m'}\right] \\
&= F_n - \lim_{\Delta\to 0} \frac{w[n+\Delta]nh + w[n+\Delta]\Delta h - \gamma w[n]nh - (1-\gamma)w[n+\Delta]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c} J^{m'}\right] \\
&= F_n - \lim_{\Delta\to 0} \frac{w[n+\Delta]\Delta h - \gamma w[n]nh + \gamma w[n+\Delta]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c} J^{m'}\right] \\
&= F_n - \lim_{\Delta\to 0} w[n+\Delta]h - \lim_{\Delta\to 0} \gamma \frac{w[n+\Delta] - w[n]}{\Delta} nh + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c} J^{m'}\right] \\
&= F_n - w[n]h - \gamma \frac{\partial w[n]}{\partial n} nh + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c} J^{m'}\right]
\end{aligned}$$

Solutions to the bargaining problem with a marginal worker

Now, we turn to the bargaining problem which is the same as standard Nash bargaining given the marginal value of employment for the worker and the marginal value of an additional employee to the firm.

$$
\begin{aligned}
\Omega^m &= wh - b - \frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + (1-\chi-p)\beta E\left[\frac{u'_c}{u_c}\Omega^{m'}\right] \\
J^m &= F_n - wh - \gamma\frac{\partial w}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]
\end{aligned}
$$

Given the bargaining power of the marginal worker ($\mu \in [0,1]$) and the bargaining powers of existing workers ($\gamma \in [0,1]$), wages and hours per worker are determined via the following standard bargaining problem.

$$
\begin{aligned}
(w,h) &= arg\max_{w,h}(\Omega^m)^\mu (J^m)^{1-\mu} \\
&= arg\max_{w,h}\left(wh - b - \frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + (1-\chi-p)\beta E\left[\frac{u'_c}{u_c}\Omega^{m'}\right]\right)^\mu \\
&\times \left(F_n - wh - \gamma\frac{\partial w}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]\right)^{1-\mu}
\end{aligned}
$$

The first order condition with respect to w gives the following sharing rule.

$$
\mu J^m = (1-\mu)\Omega^m
$$

By plugging the definitions of Ω^m and J^m into the above equation, we have

$$
\mu\left(F_n - wh - \gamma\frac{\partial w}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]\right) = (1-\mu)\left(wh - b - \frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + (1-\chi-p)\beta E\left[\frac{u'_c}{u_c}\Omega^{m'}\right]\right)
$$

It can be rewritten as

$$
\begin{aligned}
wh &= \mu\left(F_n - \gamma\frac{\partial w}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]\right) + (1-\mu)\left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b - (1-\chi-p)\beta E\left[\frac{u'_c}{u_c}\Omega^{m'}\right]\right) \\
&= \mu\left(F_n - \gamma\frac{\partial w}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right] - (1-\chi-p)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]\right) + (1-\mu)\left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b\right) \\
&= \mu\left(F_n - \gamma\frac{\partial w}{\partial n}nh + p\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]\right) + (1-\mu)\left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b\right) \\
&= \mu\left(F_n - \gamma\frac{\partial w}{\partial n}nh + p\frac{\kappa}{q}\right) + (1-\mu)\left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b\right) \\
&= \mu\left(F_n - \gamma\frac{\partial w}{\partial n}nh + \frac{V}{1-N}\kappa\right) + (1-\mu)\left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b\right)
\end{aligned}
$$

The sharing rule ($\mu J^m = (1-\mu)\,\Omega^m$) is used in the second line and the optimal condition for vacancies ($\kappa = qE\left[\widetilde{\beta}J^{m'}\right] = q\beta E\left[\frac{u_c'}{u_c}J^{m'}\right]$) is used in the third line. The definitions of p and q are used in the last line.

The first order condition with respect to h gives the following intra-temporal condition for hours per worker.

$$\mu J^m\left(w - \frac{\widetilde{u}_l(1-h)}{u_c}\right) = (1-\mu)\Omega^m\left(-\frac{\partial F_n}{\partial h} + w + \gamma\frac{\partial w}{\partial n}n\right)$$

Since $\mu J^m = (1-\mu)\,\Omega^m$ holds from the first order condition with respect to w,

$$\begin{aligned}
w - \frac{\widetilde{u}_l(1-h)}{u_c} &= -\frac{\partial F_n}{\partial h} + w + \gamma\frac{\partial w}{\partial n}n \\
\frac{\widetilde{u}_l(1-h)}{u_c} &= \frac{\partial F_n}{\partial h} - \gamma\frac{\partial w}{\partial n}n \\
&= \frac{\partial}{\partial h}\left(\frac{\partial F(z,k,nh)}{\partial n}\right) - \gamma\frac{\partial w}{\partial n}n \\
&= \frac{\partial}{\partial h}\left(F_{nh}h\right) - \gamma\frac{\partial w}{\partial n}n \\
&= F_{nh} - \gamma\frac{\partial w}{\partial n}n
\end{aligned}$$

Following Andolfatto (1996) and Cheron and Langot (2004), it is assumed that a weight of each worker is small so that F_{nh} is taken as given by both the worker and the firm during the wage bargaining.

Solutions to the first order differential equation with respect to wages

Given the Cobb-Douglas production function, the sharing rule, and the intra-temporal condition, the wage bill can be written as

$$\begin{aligned}
\mu J^m &= (1-\mu)\,\Omega^m \\
\frac{\widetilde{u}_l(1-h)}{u_c} &= (1-\alpha)\,zk^\alpha\,(nh)^{-\alpha} - \gamma\frac{\partial w}{\partial n}n \\
wh &= \mu\left((1-\alpha)\,zk^\alpha h^{1-\alpha}n^{-\alpha} - \gamma\frac{\partial w}{\partial n}nh + \frac{V}{1-N}\kappa\right) + (1-\mu)\left(\frac{\widetilde{u}\,(1) - \widetilde{u}\,(1-h)}{u_c} + b\right)
\end{aligned}$$

We can rewrite the wage bill as the first order differential equation as follows

$$\begin{aligned}
\mu\gamma nh\frac{\partial w}{\partial n}+hw &= \mu\left((1-\alpha)\,zk^{\alpha}h^{1-\alpha}n^{-\alpha}+\frac{V}{1-N}\kappa\right)+(1-\mu)\left(\frac{\widetilde{u}(1)-\widetilde{u}(1-h)}{u_c}+b\right)\\
\frac{\partial w}{\partial n}+\frac{1}{\mu\gamma n}w &= \frac{1}{\mu\gamma nh}\left(\mu\left((1-\alpha)\,zk^{\alpha}h^{1-\alpha}n^{-\alpha}+\frac{V}{1-N}\kappa\right)+(1-\mu)\left(\frac{\widetilde{u}(1)-\widetilde{u}(1-h)}{u_c}+b\right)\right) \quad (48)
\end{aligned}$$

Therefore, the integrating factor is

$$e^{\int\left(\frac{1}{\mu\gamma n}\right)dn}=e^{\frac{1}{\mu\gamma}ln(n)}=n^{\frac{1}{\mu\gamma}}$$

By multiplying both sides of Equation (44) by $n^{\frac{1}{\mu\gamma}}$ and integrating both sides with respect to n, we have

$$\begin{aligned}
w &= n^{-\frac{1}{\mu\gamma}}\int n^{\frac{1}{\mu\gamma}}\frac{1}{\mu\gamma nh}\left[\mu\left((1-\alpha)\,zk^{\alpha}h^{1-\alpha}n^{-\alpha}+\frac{V}{1-N}\kappa\right)+(1-\mu)\left(\frac{\widetilde{u}(1)-\widetilde{u}(1-h)}{u_c}+b\right)\right]dn+Dn^{-\frac{1}{\mu\gamma}}\\
&= n^{-\frac{1}{\mu\gamma}}\int n^{\frac{1}{\mu\gamma}}\frac{1}{\mu\gamma nh}\left[\mu\left((1-\alpha)\,zk^{\alpha}h^{-\alpha}n^{-\alpha}\right)+\mu\left(\frac{V}{1-N}\kappa+\frac{1-\mu}{\mu}\left(\frac{\widetilde{u}(1)-\widetilde{u}(1-h)}{u_c}+b\right)\right)\right]dn+Dn^{-\frac{1}{\mu\gamma}}\\
&= n^{-\frac{1}{\mu\gamma}}\left[\int\frac{(1-\alpha)}{\gamma}zk^{\alpha}h^{-\alpha-1}n^{-\alpha-1+\frac{1}{\mu\gamma}}dn+\int n^{\frac{1}{\mu\gamma}-1}\frac{1}{\gamma h}\left(\frac{V}{1-N}\kappa+\frac{1-\mu}{\mu}\left(\frac{\widetilde{u}(1)-\widetilde{u}(1-h)}{u_c}+b\right)\right)dn\right]+Dn^{-\frac{1}{\mu\gamma}}\\
&= n^{-\frac{1}{\mu\gamma}}\left[\frac{\mu(1-\alpha)}{1-\mu\gamma\alpha}zk^{\alpha}h^{-\alpha}n^{-\alpha+\frac{1}{\mu\gamma}}+n^{\frac{1}{\mu\gamma}}\frac{\mu}{h}\left(\frac{V}{1-N}\kappa+\frac{1-\mu}{\mu}\left(\frac{\widetilde{u}(1)-\widetilde{u}(1-h)}{u_c}+b\right)\right)\right]+Dn^{-\frac{1}{\mu\gamma}}\\
&= \frac{\mu(1-\alpha)}{1-\mu\gamma\alpha}zk^{\alpha}h^{-\alpha}n^{-\alpha}+\frac{\mu}{h}\left(\frac{V}{1-N}\kappa+\frac{1-\mu}{\mu}\left(\frac{\widetilde{u}(1)-\widetilde{u}(1-h)}{u_c}+b\right)\right)+Dn^{-\frac{1}{\mu\gamma}}
\end{aligned}$$

where D is a constant of the integration of the homogeneous equation. By assuming the total wage bill (wnh) has to remain finite as employment becomes small as in Hawkins (2011) or alternatively by assuming $\lim_{n\to 0} wnh=0$ as in Cahuc et al. (2008), we have $D=0$. From the equation above, we also have

$$\frac{\partial w}{\partial n}=-\frac{\mu\alpha(1-\alpha)}{1-\mu\gamma\alpha}zk^{\alpha}h^{-\alpha}n^{-\alpha-1}<0$$

Equilibrium conditions

The equilibrium of the model is characterized by the following conditions under functional forms specified in the calibration section.

$$
\begin{aligned}
E\left[\beta\frac{C}{C'}\left(1+r'\right)\right] &= 1 \\
r &= \alpha\frac{Y}{K}-\delta \\
N^{'} &= (1-\chi)N+\omega V^{\psi}\left(1-N\right)^{1-\psi} \\
q &= \omega V^{\psi-1}\left(1-N\right)^{1-\psi} \\
Y &= C+I+\kappa V \\
I &= K^{'}-\left(1-\delta\right)K \\
Y &= e^{z}K^{\alpha}\left(Nh\right)^{1-\alpha} \\
\frac{\kappa}{q} &= E\left[\beta\frac{C}{C'}\left[\frac{1-\alpha}{1-\mu\gamma\alpha}\frac{Y^{'}}{N'}-w^{'}h^{'}+\left(1-\chi\right)\frac{\kappa}{q'}\right]\right] \\
\phi_{e}\left(1-h\right)^{-\eta}C &= \frac{\left(1-\alpha\right)}{1-\mu\gamma\alpha}\frac{Y}{Nh} \\
wh &= \mu\left(\frac{1-\alpha}{1-\mu\gamma\alpha}\frac{Y}{N}+\frac{V}{1-N}\kappa\right)+\left(1-\mu\right)\left(\left(\phi_{e}\frac{1}{1-\eta}-\phi_{e}\frac{\left(1-h\right)^{1-\eta}}{1-\eta}\right)C+b\right)
\end{aligned}
$$

Data Appendix

Raw data

- Current Population Survey (CPS) and Current Employment Statistics (CES): Bureau of Labor Statistics (BLS)

Population (from CPS), employment (from CPS), average weekly hours (production and non-supervisory employees, total private, from CES)

- National Income and Product Accounts (NIPA): Bureau of Economics Analysis (BEA)

Gross domestic product (GDP), gross national product (GNP), personal consumption expenditures (durable goods, nondurable goods, services), gross private domestic investment (fixed investment), consumption of fixed capital (CFC), statistical discrepancy (SD), compensation of employees (CE), proprietors' income (PI), rental income (RI), corporate profits (CP), net interest (NI), taxes on production and imports (Tax), subsidies (Sub), business current transfer payments (BCTP), current surplus of government enterprises (CSGE), GDP deflator (price index for GDP)

- Composite Help-Wanted Index constructed by Barnichon (2010)

Monthly data for vacancies (composite help-wanted index) are available up to 2016m12.

Constructed data

All variables are quarterly and seasonally adjusted. Nominal variables are converted to real variables using the GDP deflator.

- Output (Y) = real GDP
- Consumption (C) = real personal consumption expenditures (nondurable goods and services)

- Investment (I) = real gross private domestic investment (fixed investment) and real personal consumption expenditures (durable goods)
- Depreciation (Dep) = real consumption of fixed capital
- Capital Stocks (K) is constructed by the perpetual inventory method using the following law of motion:

$$K_{t+1} = (1-\delta)K_t + I_t$$

The initial capital stock ($K_{1947:Q1}$) and the quarterly depreciation rate of capital stocks (δ) are jointly chosen satisfying the following two conditions:

1. The capital-output ratio ($\frac{K}{Y}$) does not display any trend over the period 1947:Q1-1956:Q4

$$\frac{K_{1947:Q1}}{Y_{1947:Q1}} = \frac{1}{40} \sum_{t=1947:Q1}^{1956:Q4} \frac{K_t}{Y_t}$$

2. The average ratio of depreciation to output using constructed series of capital stocks matches the average ratio of depreciation to output in the U.S. data over the periods 1947:Q1-2018:Q4

$$\frac{1}{288} \sum_{t=1947:Q1}^{2018:Q4} \frac{\delta K_t}{Y_t} = 0.0348 \left(= \frac{1}{288} \sum_{t=1947:Q1}^{2018:Q4} \frac{Dep_t}{Y_t} \right)$$

- Employment = employment in CPS
- Hours per worker = $\frac{\text{average weekly hours in CES}}{20 \times 5}$
- Total hours = employment × hours per worker
- Labor share is computed following Ríos-Rull and Santaeulalia-Llopis (2010)

Labor share is defined as $1 - \frac{\text{capital income (CI)}}{\text{GNP}}$. The capital income (CI) consists of unambiguous capital income (UACI) and ambiguous capital income (ACI). The unambiguous capital income (UACI) is composed of rental income (RI), corporate profits (CP), net interest (NI), and current surplus of government enterprises (CSGE). In order to compute ambiguous capital income (ACI), it is assumed that the proportion of unambiguous capital income (UACI) to unambiguous income (UAI), denoted by θ, is the same as the proportion of ambiguous capital income (ACI) to ambiguous income (AI). For each period, the proportions (θ) of unambiguous capital income (UACI) to unambiguous income (UAI) are computed as follows:

$$\theta = \frac{UACI}{UAI} = \frac{UACI}{CE + CFC + UACI} = \frac{RI + CP + NI + CSGE}{CE + CFC + (RI + CP + NI + CSGE)}$$

Then, the proportions (θ) are used to compute the ambiguous capital income (ACI) as follows:

$$\theta = \frac{UACI}{UAI} = \frac{ACI}{AI} \quad \Rightarrow \quad ACI = \theta AI$$

Note that ambiguous income (AI) consists of proprietors' income (PI), taxes on production and imports (Tax) less subsidies (Sub), business current transfer payments (BCTP), and statistical discrepancy (SD). Finally, labor share can be computed as follows:

$$labor\ share = 1 - \frac{CI}{GNP} = 1 - \frac{UACI + ACI}{GNP} = 1 - \frac{UACI + \theta AI}{GNP}$$

- Real wage $= \frac{\text{labor share} \times \text{output}}{\text{total hours}}$

- Labor productivity $= \frac{\text{output}}{\text{total hours}}$

Output, consumption, investment, capital stocks, and employment are normalized by population, logged and HP filtered with a smoothing parameter of 1600 when I compute business cycle statistics.